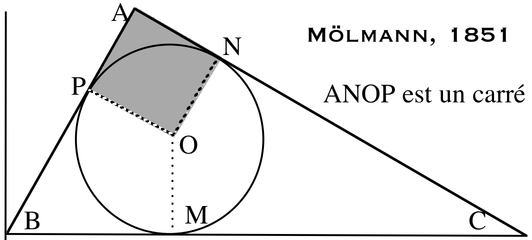
LE THEOREME DE PYTHAGORE

quelques démonstrations

cf : "Curiosités géométriques
E.Fourrey (Vuibert, 1938)
Le Théorème de Pythagore
(IREM Paris-Nord, 1980)

Mölmann, 1851



ANOP est un carré

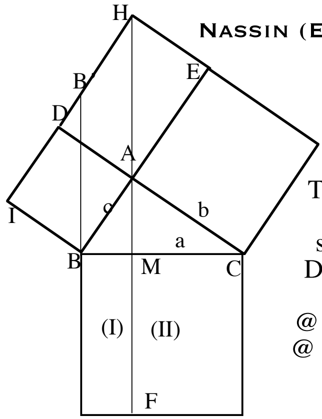
BP = BM CM = CN donc $b + c - a = 2r$

$(b + c + a)(b + c - a) = (b + c + a)\,2r$
$= 4\,@(ABC) = 2\,bc$

$(b + c + a)(b + c - a) = (b + c)^2 - a^2$
$(b + c)^2 - a^2 = 2\,bc$

$a^2 = b^2 + c^2$

Nassin (Ed.Din, 1594)

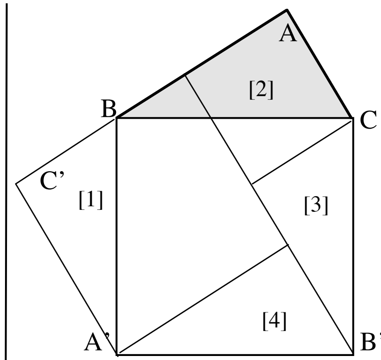


Triangles ABC, AEH, ADH superposables
Donc AH = BC

@ (BAHB') = [I]
@ (BAHB') = [II]

$= c^2$

Donc $a^2 + b^2 = c^2$



Les triangles [1], [2], [3], [4] sont égaux

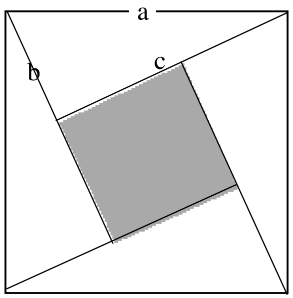
Aire pentagone ACB'A'C' =

$b^2 + c^2 + 2\ \text{Tr}$

$= a^2 + 2\ \text{Tr}$

Donc $a^2 + b^2 = c^2$

Bhaskara (XII°s.)



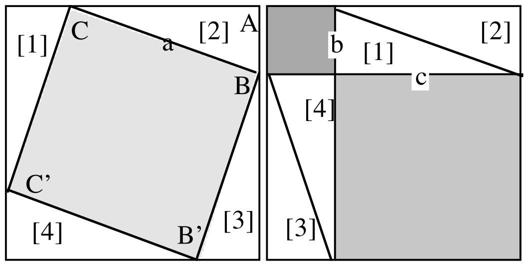
La figure centrale est un carré

$a^2 = 2bc + (c-b)^2$

$a^2 = 2bc + c^2 + b^2 - 2bc$

$a^2 = b^2 + c^2$

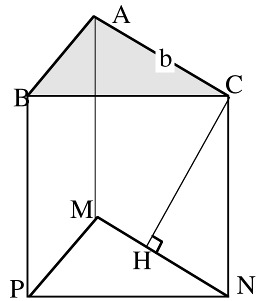
Simson (1766)



Tous les triangles rectangles sont égaux.
BCC'B' est un carré
Par différence :

$a^2 = b^2 + c^2$

P.Fabre (1888)



Les triangles CHN, MNP, ABC sont égaux

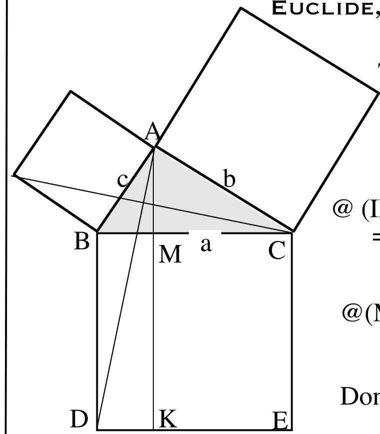
Donc CH = MN = b

$@\,(ACNM) = b^2$

$@\,(ABPM) = c^2$

$a^2 = b^2 + c^2$

Euclide, (3°s av.J-C.)



Triangles ABD et IBC superposables

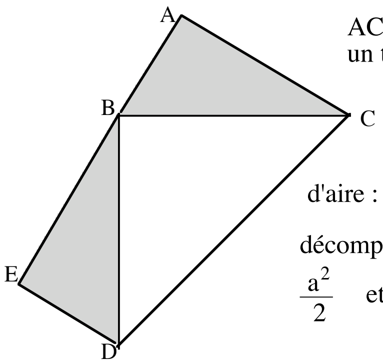
$a\,(IBC) = c^2/2$

@ (IBC) = a (ABD)
= @ (BMKD)/2
de même :

$@(MCEK) = b^2/2$

Donc $a^2 + b^2 = c^2$

Garfield 1882



ACDE est un trapèze

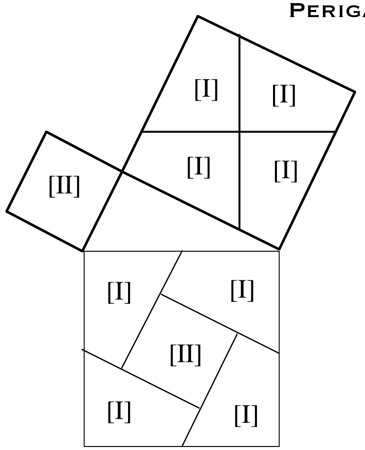
d'aire : $\frac{(b+c)^2}{2}$

décomposée en :

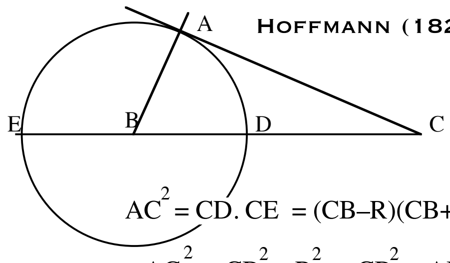
$\frac{a^2}{2}$ et $\frac{2\,bc}{2}$

$(b+c)^2 = a^2 + 2bc$

Perigal (1873)

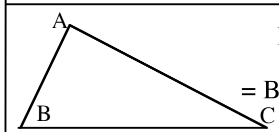


Hoffmann (1821)



$AC^2 = CD.\,CE = (CB-R)(CB+R)$

$AC^2 = CB^2 - R^2 = CB^2 - AB^2$

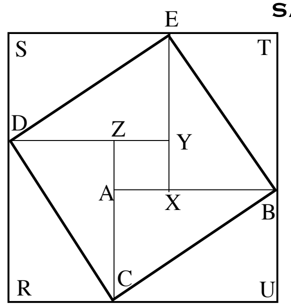


$BC^2 = (\overrightarrow{BA} + \overrightarrow{AC})^2$

$= BA^2 + AC^2 + 2\,\overrightarrow{BA}.\overrightarrow{AC}$

$BC^2 = AB^2 + AC^2$

Sauderson (~1739)



$@\,(AZYB) = (c-b)^2$
$@\,(DEBC) = a^2$
$@\,(RSTU) = (c+b)^2$

$a^2 = (c+b)^2 - 4\,t[ABC]$
$a^2 = (c-b)^2 + 4\,t[ABC]$
$2\,a^2 = (c+b)^2 + (c-b)^2$
$= 2b^2 + 2c^2$

N.B. : @ ( ) désigne l'aire d'une surface

F.Boule,